

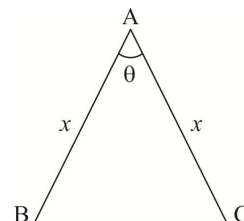
Solutions to Maths workbook – 1 | Trigonometric Identities and Equations

JEE Advanced Archive

Daily Tutorial Sheet - 1

1.(B) Area of $\Delta = \frac{1}{2}x^2 \sin \theta$ which is maximum when $\sin \theta = 1 \Rightarrow \theta = \frac{\pi}{2}$

$\therefore$ Maximum area $= \frac{1}{2}x^2$



2.(B) $\sin x + \cos x = \frac{1}{2} \Rightarrow (1 + \tan x)^2 = \sec^2 x \Rightarrow 4(1 + \tan^2 x + 2 \tan x) = \sec^2 x$
 $\Rightarrow 4 + 4 \tan^2 x + 8 \tan x = 1 + \tan^2 x \Rightarrow 3 \tan^2 x + 8 \tan x + 3 = 0$
 $\Rightarrow \tan x = \frac{-8 \pm \sqrt{28}}{6} = \frac{-4 \pm \sqrt{7}}{3} \therefore \tan x = \frac{-(4 + \sqrt{7})}{3}$

3.(C) $\tan\left(\frac{P}{2}\right) + \tan\left(\frac{Q}{2}\right) = \frac{-b}{a}$ and $\tan\left(\frac{P}{2}\right) \tan\left(\frac{Q}{2}\right) = \frac{c}{a}$

Now $P + Q = \frac{\pi}{2} \Rightarrow \frac{P}{2} + \frac{Q}{2} = \frac{\pi}{4} \Rightarrow \tan\left(\frac{P}{2} + \frac{Q}{2}\right) = 1 \Rightarrow \frac{\tan\left(\frac{P}{2}\right) + \tan\left(\frac{Q}{2}\right)}{1 - \tan\left(\frac{P}{2}\right) \tan\left(\frac{Q}{2}\right)} = 1$

$\Rightarrow \frac{-\frac{b}{a}}{1 - \frac{c}{a}} \Rightarrow c = a + b$

4.(C) $\frac{\sqrt{3} \sin x + \cos x}{\sin x \cos x} + \frac{2(\sin^2 x - \cos^2 x)}{\sin x \cos x} = 0$

$\Rightarrow \sqrt{3} \sin x + \cos x = 2(\cos^2 x - \sin^2 x) \Rightarrow \frac{\sqrt{3}}{2} \sin x + \frac{1}{2} \cos x = \cos 2x$

$\Rightarrow \cos\left(x - \frac{\pi}{3}\right) = \cos 2x \Rightarrow x - \frac{\pi}{3} = 2n\pi \pm 2x \Rightarrow x \pm 2x = 2n\pi + \frac{\pi}{3}$

$\Rightarrow x = \frac{2n\pi}{3} + \frac{\pi}{9}$ or $-2n\pi - \frac{\pi}{3}$

As $x \in (-\pi, \pi)$; So $x = \frac{-\pi}{3}, \frac{\pi}{9}, \frac{7\pi}{9}$ and $\frac{-5\pi}{9} \Rightarrow$ Sum of values of $x = 0$

5.(D) $\sin x + 2 \sin 2x - \sin 3x = 2 \sin 2x + 2 \sin(-x) \cos 2x = 3$

$\Rightarrow \sin 2x - (\sin x) \cos 2x = \frac{3}{2}$

$\Rightarrow -\sqrt{1 + \sin^2 x} \leq LHS \leq \sqrt{1 + \sin^2 x} \leq \sqrt{2} < \frac{3}{2}$

$\Rightarrow$ No solution

6.(D) $P: \tan \theta = \sqrt{2} + 1$ and $Q: \tan \theta = \frac{1}{\sqrt{2} - 1} = \sqrt{2} + 1$, $\therefore P = Q$

7.(C) $2 \sin^2 \theta - \cos 2\theta = 0 \Rightarrow \sin^2 \theta = \frac{1}{4} \Rightarrow \theta = n\pi \pm \frac{\pi}{6} \dots(i)$

Also, $2 \sin^2 \theta + 3 \sin \theta - 2 = 0 \Rightarrow (\sin \theta + 2)(2 \sin \theta - 1) = 0$

$\Rightarrow \sin \theta = \frac{1}{2} \Rightarrow \theta = m\pi + (-1)^m \frac{\pi}{6} \dots(ii)$

As $\theta \in [0, 2\pi]$, so common solutions of equation (i) and (ii) are: $x = \frac{\pi}{6}$ and $\frac{5\pi}{6}$

8.(A) $2 \sin^2 \theta - 5 \sin \theta + 2 > 0 \Rightarrow (2 \sin \theta - 1)(\sin \theta - 2) > 0$

We know that $\sin \theta \leq 1 \Rightarrow \sin \theta - 2 < 0$

$\Rightarrow (2 \sin \theta - 1) < 0 \therefore \theta \in \left(0, \frac{\pi}{6}\right) \cup \left(\frac{5\pi}{6}, 2\pi\right)$

9.(C) $(\cos x + \cos 3x) + (\cos 2x + \cos 4x) = 0$

$\Rightarrow 2 \cos 2x \cos x + 2 \cos 3x \cos x = 0 \Rightarrow \cos x(\cos 2x + \cos 3x) = 0$

$\Rightarrow \cos x \cos \frac{5x}{2} \cos \frac{x}{2} = 0 \therefore x = (2n+1)\frac{\pi}{2}, 2(2m+1)\frac{\pi}{2} \text{ or } \frac{2}{5}(2k+1)\frac{\pi}{2}$

So, there are 7 possible values of x : $\frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{5}, \frac{3\pi}{5}, \pi, \frac{7\pi}{5}, \frac{9\pi}{5}$

10.(A) $(\sin \theta + \sin 7\theta) + \sin 4\theta = 0 \Rightarrow 2 \sin 4\theta \cos 3\theta + \sin 4\theta = 0 \Rightarrow (\sin 4\theta)(2 \cos 3\theta + 1) = 0$

$\Rightarrow \sin 4\theta = 0 \text{ or } \cos 3\theta = \frac{-1}{2} \Rightarrow \theta = \frac{n\pi}{4} \text{ or } \theta = \frac{2k\pi}{3} \pm \frac{2\pi}{9}$

$\Rightarrow \theta = \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \frac{2\pi}{9}, \frac{8\pi}{9}, \frac{4\pi}{9}$

11.(D) $(\sin x + 3)(2 \sin x - 1) = 0 \Rightarrow \sin x = \frac{1}{2} [\because \sin x \neq -3] \dots(i)$

$\therefore$ There are 4 values of x ; $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}$ and $\frac{17\pi}{6}$

Such that $x \in [0, 3\pi]$ and satisfying equation (i)